PHYSICS AND THE STUDY OF COMPLEX SYSTEMS

Detailed syllabus and some reading material.

Note: the reading part may be updated based on your questions/personal interests.

Part I: Complexity in systems with large numbers of uncorrelated random variables.

Chapter 1: PROBABILITIES

0 Probabilities: the frequentist and Bayesian points of view. The nature of randomness.

1 Discrete and continuous probabilities. Moments, cumulants, characteristic functions.

2 Normal and log-normal distributions. Breakdown of cumulant expansion and a first look at tail effects.

3 Central limit theorem and saddle point methods. Large deviations.

4 The renormalization group approach to CLT. First notions of scale covariance.

Basic reading: Feller, chapters 1-5; Bouchaud and Potters, chapter 1. For the CLT see the cute applet[1] https://digitalfirst.bfwpub.com/stats_applet/stats_applet_3_cltmean.html. Wikipedia articles on Bayesian inference, normal and log normal distributions, and martingales, are all good and useful. Look up also an article on Log-normal Distributions E. Limpert, W. A. Stahel and M. Abbt, Bioscience, Vo. 51, 2001.

Chapter 2: RANDOM WALKS/BROWNIAN MOTION

1 Random walks in the stock market: the efficient market hypothesis

2 Discrete Brownian motion. Returns to the origin, counter intuitive aspects.

3 An example : early models of stock returns. The Cowles and Jones test.

[1] For technical reasons, some of these links are not clickable, or send you to the wrong page. In this case, please type them manually in your browser address window. Also, some of the links require using Java. This can be a challenge depending on your machine and your browser- feel free to ask me for advice. I recommend using Chrome and
https://chrome.google.com/webstore/detail/cheerpj-applet-runner/bbmolahhldcbngedljfadjlognfaaein/rel

4 Continuous Brownian motion. Path integrals.

5 Relation with quantum mechanics. Fokker Planck equations and Schrödinger equation.

6 Dynamics with random forces: Langevin equation. White, pink, brown and black noise.

Basic reading: Feller, chapter 4; Campbell et al., chapter 2. For efficient market hypothesis, look at https://www.fxcm.com/markets/insights/is-the-efficient-market-hypothesis-still-valid/

Advanced Reading:

For Langevin and Fokker Planck equations: look at N.G. Van Kampen, "Stochastic processes in physics and chemistry", North Holland, Amsterdam (1981). For a detailed discussion of Ito's calculus in relation with path integrals and quantum mechanics, look at the article by the same author in J. Stat. Phys. 24 (1981) 175.

For continuous Brownian motion in relation with quantum mechanics: look also at the textbook by R. P. Feynman, "Statistical Mechanics" , Benjamin (1998).

Digression 1: PARADOXICAL GAMES

1 Parrondos's paradoxical games

2 Brownian ratchets and biology.

Basic and Advanced Reading: look up Parrondo's web page http://seneca.fis.ucm.es/parr. You can play with flashing ratchets at https://elmer.unibas.ch//bm. You can find applications of Parrondo's paradox to Poker at http://parrondoparadox.blogspot.com/2012/07/parrondos-paradox-poker.html.

Chapter 3: OPTIONS PRICING: THE BLACK AND SCHOLES FORMULA

1 Generalities: options problems and evolution of wealth.

2 Formalization. The option price.

3 Risk minimization. The bank's strategy and the Black and Scholes formula.

4 Extensions and generalizations. What is risk?

5 Aspects of stock market statistics.

Basic Reading: Voit, chapters 2 and 4. Bouchaud and Potters, chapter 4.

Advanced Reading:

For the Black and Scholes formula and the options problem considered as a physics problem, look at: J .P. Bouchaud, D. Sornette, J. Phys. I. 4 (1994) 863; ibid 5 (1995) 219. Look also at the letter to Physics Today by these authors, in the volume 128, March 1996, p. 15. For a popular article criticizing the Black and Scholes pricing method, see https://www.theguardian.com/science/2012/feb/12/black-scholes-equation-credit-crunch.

Digression 2: EQUILIBRIA

1 Pareto efficiency

2 Nash equilibria.

Reading: look up definitions on Wikipedia.

Chapter 4: LONG TAILS, GENERALIZED CENTRAL LIMIT THEOREMS

1 Tail effects, Levy laws in physics and elsewhere.

2 Statistics of extremes. Generalized central limit theorems.

3 Truncated Levy laws. Scaling in earthquakes. Scaling in stock prices.

Basic Reading: Schroeder, chapter 6; Bouchaud and Potters, chapter 1, Voit, chapter 5.

Advanced Reading:

For Levy laws: G. Zaslavsky, U. Frisch, M. Schlesinger, "Levy flights and related topics in physics", Lecture Notes in Physics, 450, Springer, 1995; and I. Koponen, Phys. Rev. E52 (1995) 1197.

For truncated Levy laws in financial markets: R. Mantegna, H. E. Stanley, Nature 376 (1995) 46.

For the Gutenberg Richter law and other aspects of earthquakes (Chapters 4,8,9), you can look at the book by D.L. Turcotte, "Geodynamics: application of continuum physics to geological problems", Wiley (1992).

For the problem of statistics of extremes there is a nice, physics oriented chapter devoted to that subtle question in J.P. Bouchaud and A. Georges, "Anomalous diffusion in disordered media", Physics Reports 195 (1990).

Chapter 5: STATISTICAL APPROACH TO HYPOTHESIS TESTING

1 The example of a loaded die. Theory, experiment, and modification of theory.

Basic Reading: J. Rau, "Statistical mechanics in a nutshell", http://arxiv.org/abs/physics/9805024.

Advanced Reading:

For statistical aspects of hypothesis testing : look at Jaynes's analysis of a biased die, E.T. Jaynes, "Concentrations of distributions at entropy maxima", in E. T. Jaynes, Papers on Probability, Statistics and Statistical Mechanics, ed R. D. Rosenkrantz, Kluwer Academic, Dordrecht (1989).

Part II: Correlations, complexity and catastrophes.

Chapter 6: PROBABILITIES AND THERMODYNAMICS

1 Entropy and information.

2 The basic principle. Microcanonical, canonical and grand canonical distributions.

3 Large systems, fluctuations and thermodynamic limit.

4 Application to complex systems: optimization problems and simulated annealing.

Basic Reading: Robertson, chapters 2-4.
Advanced Reading:

For more classical thermodynamics: the book "Thermodynamics and an Introduction to Thermostatistics" H.B.Callen, Wiley (1960), is a rare jewel.

For optimization problems, look at: S. Kirkpatrick, C. D. Gelatt and M. P. Vecchi, "Optimization by simulated annealing", Science 220 (1983) 671; and S. Kirkpatrick, "Optimization by simulated annealing: quantitative studies", J. Stat. Phys. 35 (1984) 975.

For the central limit theorem and correlated variables, look up H. Hilhorst, "Central limit theorem for correlated variables: some critical remarks", http://arxiv.org/abs/0901.1249

Chapter 7: CORRELATIONS AND CRITICAL PHENOMENA

1 Phenomenology of ferromagnetic transitions, first and second order.

2 Mean field theory; transitions, bifurcations, precursors.

3 Renormalization group; exact results on hierarchical lattices, approximate results on Euclidian lattices.

Basic Reading: Strogatz, Chapter 3; Robertson, chapter 5; Schroeder, chapter 16.

Chapter 8: SELF ORGANIZED CRITICALITY

1 Random geometry: percolation, Brownian motion again, fractals, cracks, DLA.

2 Self organized critical phenomena. Cellular automata.

3 Criticality and phases in minority games.

Basic Reading: Schroeder, Chapter 15.

Advanced Reading:

For self organized criticality, look at P. Bak, C. Tang and K. Weisenfeld, "Self organized criticality, an explanation of $1/f$ noise"Phys. Rev. A38 (1987) 364; P. Bak and C. Tang, "Earthquakes as self organized critical phenomena", J. Geophys. Res. 94 (1989) 15,635; A. Sornette and D. Sornette, "Self organized criticality and earthquakes", Europhys. Lett. 9 (1989) 197. Be wary of the claims made by the authors however. You may want to look at J. Kirchner and A. Weil, "No fractals in fossil extinciton statistics", Nature 395 (1998) to get a feeling of the dangers of claiming that everything is powerlaw, and therefore SOC.

For geometrical problems, look at the little book by D. Stauffer, "Theory of percolation", Taylor and Francis (London 1992).

For fun with cellular automata look up Conway's "Game of life" on Wikipedia and the applet at https://playgameoflife.com/

For fun with DLA look up the applet
http://www.kclee.de/clemens/java/dla/fast_dla.html
For fun with sandpiles visit
http://maslov.bioengineering.illinois.edu/Sandpile.htm

For fun with games, look up
https://rf.mokslasplius.lt/minority-game/
You can also look up the Minority Game web page
http://www.cscs.umich.edu/∼crshalizi/notabene/minority-game.html

Chapter 9: PHENOMENOLOGY OF CATASTROPHES

1 Simple rupture models. Earthquakes.

2 Can one predict stock market crashes?

Basic Reading: there is now a book, "Why stock markets crash: critical events in complex financial systems", Princeton University Press. To take with a grain of salt.

Other Reading:

For the prediction of catastrophes, look at the attempts: D. Sornette and C.G. Sammis, "Complex critical exponents from renormalization group theory of earthquakes: implications for earthquake predictions", J. Phys. I 5 (1995) 607; D. Sornette, A. Johansen and J. P. Bouchaud, "Stock market crashes, precursors and replicas", J. Phys. I 6 (1996) 1. Papers appear every day on the net about the latest ripples of the stock market. Look at recommended web sites.

Part III: Other points of view.

Chapter 10: DYNAMICAL SYSTEMS

1 Simple examples of non linear dynamics.

2 One dimensional flows, a second look at bifurcations.

3 Lyapunov exponents, attractors and chaos.

4 Is the stock market described by a low dimensional dynamical system?

Basic Reading: Schroeder, Introduction and chapter 12; Strogatz, chapter 10.

Advanced Reading:

To get more insights on chaos in physics and other natural sciences, look at the book by D. Ruelle, "Chance and chaos" (1991) Princeton. The article "Can non linear dynamics help economists" by Ruelle in "The economy as an evolving complex system", mentioned in the general references, is clear and much to the point. For lots of chaos games, look up http://math.bu.edu/DYSYS/chaos-game/node4.html

Chapter 11: RANDOM NETWORKS

1 Real networks: internet, cellula networks, protein folding etc.

2 Bits of random graph theory.

3 Generalized random graphs.

4 Small world networks/scale free networks

Basic Reading: The excellent review "Statistical mechanics of complex networks" by R. Albert and A.L Barabasi, http://arxiv.org/abs/cond-mat/0106096

Advanced Reading:

To get more insights on the role of networks in cell biology, "The large-scale organization of metabolic networks", by H. Jeong, B. Tombor,R. Albert, Z.N. Oltvai and A.L Barabasi, http://arxiv.org/abs/cond-mat/0010278

TEXTBOOKS

There is unfortunately no single good quality textbook covering the material of this course. There are however quite a few specialized books that handle very well, in some of their chapters, most of the topics covered in this course. In the syllabus, the references are to the following:

W. Feller, “An introduction to probability theory and its applications”, Wiley (1971).
J.Y. Campbell, A. W. Lo and A. C. MacKinlay, “The econometrics of financial markets”, Princeton (1997).
J.P. Bouchaud, M. Potters, “Theory of financial risk and derivatives pricing: from statistical physics to risk management”, Cambridge University Press (2002).
H.S. Robertson, “Statistical Thermophysics”, Prentice Hall (1993)
M. Schroeder, “Fractals, chaos, power laws”, Freeman (1991).
S. H. Strogatz, “Non linear dynamics and chaos”, Addison-Wesley (1997).
J. Voit, “The statistical mechanics of financial markets”, Springer (2005)